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Cruise Design for a 5-Year Period of the 50-Year Timber Sales in Alaska

John W. Hazard

Abstract

Sampling rules and estimation procedures are described for a new cruise design that was developed for 50-year timber sales in Alaska. An example is given of the rate redetermination cruise and analysis for the 1984-1989 period of the Ketchikan Pulp Company sale. In addition, methodology is presented for an alternative sampling technique of sampling with probability proportional to size, sample size calculations, and volume equation development.

Keywords: Timber cruising, sampling design, timber sales, Alaska.

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JOHN W. HAZARD is Station statistician, Forestry Sciences Laboratory, Pacific Northwest Forest and Range Experiment Station, P.O. Box 3890, Portland, Oregon 97208.

Introduction

In the 1950's, the USDA Forest Service awarded 50-year timber sale contracts to Ketchikan Pulp Co. (KPC) and Alaska Lumber and Pulp Co. (ALP) for 1.5 billion cubic feet and 4.97 billion board feet, respectively. These sales, awarded by the Tongass National Forest (Alaska Region), contain a provision for rate redeterminations every 5 years that creates a need to conduct timber cruises for determination of volume and values of included timber. The KPC sale requires that the rate redetermination appraise 960 million board feet (MMBF), and the ALP sale requires an appraisal of 633 MMBF every 5 years for the 50-year period of the sales. These large volumes make it important to use efficient cruise procedures.

The long-term sales have characteristics that complicate the cruise design:

1. The volume for each 5-year period is contained within a large group of cutting units that range in size from a few acres to over 150 acres. The KPC rate redetermination for 1984-89 began with a base of 576 cutting units encompassing 38,640 acres. The ALP 1981-86 rate redetermination contained 477 units encompassing 21,645 acres.

2. The cutting units to be harvested are determined through a negotiation process that allows input from the purchaser before and after the cruise takes place; this causes uncertainty over the selection of the final cutting units.

3. An environmental impact statement (EIS) is prepared for all cutting units in the initial selection for each 5-year period. The final environmental decision may require that some otherwise available cutting units be omitted after they are cruised; this also causes uncertainty over the final cutting units to be selected.

4. The Tongass Land Management Plan requires a specified harvest level to be taken proportionally, by acreage, from four volume classes. Shifts in acreage caused by the selection process may necessitate changes in final cutting units selected in order to meet harvest levels by volume class.

5. New blowdown areas may be substituted as cutting units as they occur. In the 1979-84 KPC rate redetermination, approximately 200 MMBF of timber blew down between cruise completion and the rate redetermination. A major volume substitution had to be made without supporting cruise data.

6. Cutting units not harvested in a preceding period may be carried over to the subsequent period. Market and other factors determine harvest levels in the last year of a period and make the number and location of carryover units uncertain.

7. Cutting unit selection is tentatively completed prior to the cruise. Upon cruise completion, the volume may be more or less than required for the period and may require the deferral or addition of units.

8. Previous cruises failed to adequately sample low-volume, high-value Alaska-cedar (*Chamaecyparis nootkatensis* [D. Don] Spach).

As a result of these problems, the Alaska Region, USDA Forest Service, in conjunction with the Pacific Northwest Forest and Range Experiment Station, designed and implemented a cruise design for the KPC and ALP sales that would address the foregoing problems and would specifically:

1. Allow additions or deletions of acreage to the sale after completion of field data collection for the cruise, without appreciably affecting the reliability of the results.

2. Allow field data collected in deleted units for volume-basal area prediction equations to be retained and used in cruise computations and other related studies.

3. Not require a field sample in every unit in the population.

4. Restrict the amount of individual tree field data collection,[1] but ensure precise volume estimation.

5. Sample high-value species with greater precision than for other species.

6. Provide increased individual tree data for describing species volumes.

7. Meet the specified precision requirements of the KPC and ALP sales.

Objectives

This paper presents a new cruise design to satisfy the stated requirements. It has been implemented by the Ketchikan Area Timber Management staff, Tongass National Forest. This design should have greater efficiency than previous designs because:

1. It takes advantage of gains in precision by stratifying cutting units into homogeneous groups.

2. It reduces travel cost by sampling only a subset of the available cutting units. (Travel cost is a major component of total cost.)

3. It uses an intensive basal area sample from units sampled in the field to provide information on individual species. (Collecting additional information on cutting units during a visit is less costly than going to new cutting units.)

4. It is designed to obtain accurate information from individual trees; information such as volumes by log grades, defect, and species. The FBS sample is controlled to obtain a fixed sample of basal area points distributed uniformly over the auxiliary variable of basal area.

5. It uses an estimation scheme designed to take advantage of all information known about the total sale to increase the precision of the total volume estimates.

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[1] Fall, buck, and scale (FBS) was the method of volume determination used in the KPC sale. Other volume methods can also be used with this cruise design.

In addition, it yields other products that will be used for future planning and management of the Ketchikan Area timber management program; for example, (1) volume per acre estimates by species and volume strata, (2) stand volume-basal area equations by species, (3) average defect percentages by species, and (4) average log grade percentages by species.

This new design is described as a stratified, random, equal probability sample of cutting units with selected units subsampled by an intensive systematic sample of basal area points to determine the basal area by species. The basal area by species at each point is converted to volume by stand volume-basal area equations generated from an additional subsample of basal area points as measured by FBS procedures.

A detailed description of the design (that is, sampling rules and estimation procedures) follows in the main text. Appendices are added to:

1. Describe a modification of the design to sample clearcut units with probability proportional to cutting unit size (pps).

2. Present the logic and equations for sample size calculations for both equal probability and pps sampling.

3. Present a practical method of constructing stand volume-basal area equations.

The entire package is illustrated with data and results of an actual 5-year-period cruise for the KPC sale.

The objectives of the cruise design are:

1. To estimate the gross and net volumes of the population (the total area of the 5-year-period sale) with a relative index of reliability of $\pm$ 10 percent at the 68 percent confidence probability.

2. To estimate the gross and net volume of the cedar strata with an index of reliability of $\pm$ 20 percent at the 68 percent confidence probability.

3. To estimate the following parameters by species without specified levels of precision: Volume by log grades, total defect in the standing tree (woods defect), scaling defect, number of logs per thousand board feet (MBF), and average diameter at breast height (d.b.h.).

The parameters in objective (3), in addition to the information in (1) and (2), are necessary for the appraisal process.

The Population

Proposed cutting units for the 5-year period are laid out on aerial photos and transferred to controlled base maps. The collection of all cutting units in the sale make up the population. The population is partitioned into homogeneous volume strata and a high-value species stratum (that is, collections of entire cutting units that are similar in their average volumes per acre or selected species composition are grouped as subpopulations). In addition, the selection of cutting units in the population must result in the total acreage of all cutting units in the sale being distributed proportionally to the acres appearing in the volume classes of the Tongass management plan.

To designate the units to be selected in each stratum, a prior estimate of the volume per acre by stratum is necessary. The acreage of each potential unit is multiplied by an estimate of mean volume per acre for the appropriate stratum and is then summed for all unit volumes within and among strata to produce a preliminary estimate of the total sale volume. If the initial estimate of the sale volume is low, cutting units must be added to each stratum. If the initial estimate is high, some units will have to be omitted. When the number of acres in the population are proportional to the acres by volume class in the management plan and the approximated total volume equals the required volume, the sampling population is fixed (that is, no units are added or subtracted until the sampling is completed). Adjustments may be required later but for the purposes of drawing the sample and making preliminary estimates the strata areas and number of units will not change.

In the KPC sale there were five strata used. The first four were volume strata, defined as follows:

Strata	Range in volume (MBF/acre)
1	8.0 — 16.55
2	16.56 — 25.25
3	25.26 — 37.40
4	37.41+

The fifth strata was an Alaska-cedar species strata. It included all units that were estimated, from aerial photographs, to contain the highest proportion of cedar volume. This was accomplished in the KPC sale by photointerpreting each unit. If cedar volume was obvious on a unit, that unit was included in the cedar stratum. The intent was to isolate a large percentage of the cedar volume into one stratum. Some cedar volume obviously will exist in the other volume strata.

The population may be stratified into the above-defined volume classes by several methods:

1. Each unit in the population of N units may be placed in one of the strata by either photocruising or photointerpretation or both. All units should not require photocruising to form homogeneous groups. Initially, photocruising is valuable to improve the accuracy of the interpretation.

2. Stand examinations may exist for some units or for stands in close proximity to units in the population. With the aid of type maps, average volumes per acre by types can be obtained from the stand exams. The volume of each unit can be approximated and placed in a volume stratum by first multiplying the acres in each type by the average volume per acre of that type and by then summing the volume of all types on the unit.

It is also possible to use a combination of these two systems. The method used on the KPC sale is described later in the example.

To determine which method does the best job of stratification, select a sample of units that have been previously examined on the ground. Apply the methods being contrasted to this sample of units. The stand exam volume should be assumed to be the actual volume. Each method discussed generates a different prediction of volume for each of the units in this sample.

Regress the actual values over the predicted values for both methods. The one with minimum residual mean square error should provide the best method of stratification.

All units in the population have to be placed in one of L strata ($L=5$, for the KPC sale). The number of units in each stratum is symbolized as N_h, where $h=1, \ldots, 5$. Units need to be arrayed by stratum with acres listed for each unit (M_{hi}). The symbol M_{hi} denotes the acres of the i^{th} unit in the h^{th} stratum. Therefore,

$$M_h = \sum_{i=1}^{N_h} M_{hi}$$

represents the total acreage in the h^{th} stratum.

For example, if the cedar stratum, stratum 5, had 54 units totaling 1,020 acres (413 ha), and the first unit on the list was 17 acres (7 ha) in size, the parameters of this stratum would be symbolized as follows:

$$\begin{aligned} h &= 5; \\ N_h &= N_5 = 54 \text{ units;} \\ M_{hi} &= M_{5,1} = 17 \text{ acres (7 ha); and} \\ M_h &= M_5 = \sum_{i=1}^{54} M_{5i} = 1{,}020 \text{ acres (413 ha)}. \end{aligned}$$

A tabulation of the population after it is fixed for sampling might appear as follows (the first column for each stratum lists the acres of each unit and the second column is the cumulative sum of the first column):

	Strata									
	1		2		3		4		5	
Unit no.	Area M_{1i}	Cumulative area ΣM_{1i}	Area M_{2i}	Cumulative area ΣM_{2i}	Area M_{3i}	Cumulative area ΣM_{3i}	Area M_{4i}	Cumulative area ΣM_{4i}	Area M_{5i}	Cumulative area ΣM_{5i}
	(Acres)	(Acres)								
1	$M_{1,1}$	$M_{1,1}$	$M_{2,1}$	$M_{2,1}$	$M_{3,1}$	$M_{3,1}$	$M_{4,1}$	$M_{4,1}$	17	17
2	$M_{1,2}$	$M_{1,1}+M_{1,2}$	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.
3	$M_{1,3}$	$M_{1,1}+M_{1,2}+M_{1,3}$								
.	etc.	etc.								
.										
.										
54										1020
.										blank
.										.
.										.
Total acres	M_1	—	M_2	—	M_3	—	M_4	—	1020	—

This tabulation represents the defined population. The next step is to draw the samples from each stratum.

The Design

There are two logical alternative schemes for sampling the population of N units: (1) stratified sampling with units drawn within stratum with pps; and (2) stratified random sampling with units drawn with equal probability. If the strata remain fixed (that is, no units are required to be added or deleted), and they vary considerably in size, pps sampling will be a more efficient method for estimating the total volume for the sale. If sample sizes in all strata are large, then the equal probability sampling design, alternative (2), with a "ratio-to-size" estimator, will compare favorably with the pps estimator (Cochran 1977).

When units are added or subtracted by stratum, the equal probability sample becomes more practical to apply. For example, if an equal number of units are added and subtracted from a stratum, the probability of selection of individual units under equal probability sampling remains the same. Under pps sampling the probability of selection depends on the size of each unit; thus, it will most likely change if units are substituted. Also, if units are either added or subtracted, estimation problems are easier to deal with when equal probability sampling is used.

In the cruise of the KPC sale, units were added to and subtracted from strata due to administrative decisions and catastrophic occurrences; thus, units in each stratum were sampled with equal probability. The pps sampling scheme, which should be considered when the population remains fixed, is described in the appendix (see "Probability Proportion to Size Sampling").

Small sample sizes within strata are likely to occur thereby creating problems from potential bias when using a ratio estimator. Cochran (1977) shows that the "combined ratio estimator" in stratified sampling is extremely variable, and that the "individual ratio estimator" by stratum has large bias for small samples. One of the estimation procedures that appears to provide maximum accuracy, aside from using pps sampling, is the "Quenouille ratio-type estimator." This estimation procedure is recommended by Cochran (1977) when strata are finite and sample sizes are small; it was used for the KPC sale and will be described in more detail (see "Estimation Procedures").

For simplicity, the sample size computations are based on the "individual ratio estimator" procedures. Following collection of the sample observations, the estimates are constructed using "Quenouille's estimators." These estimators are commonly referred to as "jackknife estimators." For details refer to the appendix (see "Sample Size Calculations").

Sampling Rules

Denote the sample drawn from the h^{th} stratum as n_h. In the stratified random sampling design, n_h units are drawn from N_h units with random sampling.

The samples can be drawn with replacement or without replacement. Because each unit drawn will be subsampled with an intensive grid of basal area points, the choice of method for drawing the samples may be important. If units are drawn from strata with replacement, the component of variance from variation among subsampling units (basal area points) can be ignored. This is the same situation that arises when n/N is small or N is assumed infinite. Although not necessary, it is convenient to subsample with replacement; the choice should depend on the size of the sampling fraction and the loss of precision from sampling with replacement. In the KPC sale the subsampling error component was assumed negligible.

After the number of units to be drawn is estimated, the individual units in the sample are identified. The sample for the h^{th} stratum arises from numbering the units of stratum h from 1 to N_h and drawing n_h numbers at random from this interval.

The units represented by these n_h random numbers are visited in the field and cruised for basal area by species. An intensive grid of points is constructed for each unit visited in the field. The KPC sale had one point every 3.5 acres (1.4 ha). The number of subsampling units (points) in the grid of the i^{th} unit of the h^{th} strata is denoted as m_{hi}. The number of basal area points can vary proportionally to the size of the unit, be held constant, or be arbitrary. Consideration was given to developing a self-weighting subsample in each unit. This would require the number of subsample points to be proportional to the size of the unit (that is, the subsampling fraction m_{hi}/M_{hi} is constant). This is desirable for both practical and statistical reasons and was done on the KPC sale.

To convert basal area to volume, a subsample of basal area points must be measured for volume. In the KPC sale, basal area points were sampled by fall buck and scale procedures (FBS). Separate stand volume-basal area prediction equations and log grade factors were constructed for each species. Following the completion of the field work, the point numbers and basal areas measured were arrayed by species and by basal area classes. These arrays were not constructed by strata because in a preliminary look at previous FBS data no differences were found in the volume-basal area relationships among strata.

The net and gross volume per acre prediction equations by species for the KPC sale were developed according to the detailed instructions in the appendix (see "Volume Equation Construction").

Twenty basal area points were drawn by species from each array of points in the KPC sale. The sample was uniformly distributed over the range of basal area to provide equal information for all basal area classes. Points were drawn at random within each species basal area class.

After drawing the 20 FBS plots for each species, the plots were field sampled. The following information was measured and recorded for each tree:

species,
d.b.h.,
gross volume,
net volume,
volume by log grade,
woods defect,
scaling defect, and
number of logs.

The gross and net volumes per acre by species were fitted by regression analysis to basal area per acre by species. Both linear and nonlinear models need to be investigated in both arithmetic and logarithmic scales (see "Volume Equation Construction" in the appendix).

Estimation Procedures

The detailed estimators will follow later in this section. These introductory paragraphs will describe the steps required to form estimates of total volume, volume by species, and volume by log grade. Estimates of the variance are provided only for the total volume estimates.

Picture a two-dimensional array with log grade classes as headings across the top and species as headings down the left side. Totals for the sums of all species and log grades are in the far right column and bottom row of the matrix, respectively. The grand total is the sum of the column totals or of the row totals. Refer to tables 1 and 2 in the appendix for illustrations of this matrix. A matrix with this format is constructed for the total volume (and volume per acre) for each unit sampled for basal area.

Arrays with this same format are then constructed for each strata by combining the matrices from all units sampled within the respective strata. This includes combining the species and log grade totals and the grand total. In the KPC sale, five matrices (one for each stratum) were constructed for volume per acre and five for total volume.

Estimates for the entire population are formed by multiplying the average volume per acre for each cell in the matrix by the acres in the respective stratum and summing over all strata.

The final matrix provides independent estimates for each cell that do not necessarily add to the species or log grade totals or the grand total in the array described above. The grand total is the estimate on which the precision of the cruise is controlled; therefore, if it is necessary for the individual estimates to equal the grand total, the matrix must be adjusted. This can be done by developing a matrix of proportions that expresses the total volume of each cell as a percent of the sum of the total volume of all cells. These proportions can then be applied to the independent estimates of the grand total for each variable of interest. This operation was performed on the KPC sale data.

The volumes per acre, by species for each point within the units sampled for basal area, were obtained by indirect estimation using the stand volume-basal area equations developed from the FBS points. The species volumes per point in each log grade were calculated with an average factor because a significant relationship did not exist between volume and basal area by log grade.

The volume per acre by species on each point was summed for all points on that unit and divided by the number of points to get the average volume per acre for each unit.

The volume per acre for a stratum is a ratio estimate. It is a ratio of the sum of the volumes of all units in the stratum divided by the sum of the acreage of the units. The total population volume is the sum of the stratum volumes.

The estimate of the variance of the total volume is formed as the sum of the variances of all strata weighted by the square of the strata sizes. The variance of basal area points within units (subsampling units) can be assumed to be negligible if strata are large or are sampled with replacement. The volume estimates of each basal area point are assumed to be the true volumes per acre (that is, the volume-basal area equations are treated as volume tables, and standard errors of the equations are ignored).

The estimated total volume ($\hat{Y}_R$) is given symbolically by:

$$\hat{Y}_R = \sum_{h=1}^{L} M_h \hat{R}_h = \sum_{h=1}^{L} M_h \frac{\sum_{i=1}^{n_h} \hat{Y}_{hi}}{\sum_{i=1}^{n_h} M_{hi}}; \qquad (1)$$

where:

$M_h = \sum_{i=1}^{N_h} M_{hi}$ is the total acreage in the h^{th} stratum;

$\hat{R}_h = \frac{\sum_{i=1}^{n_h} \hat{Y}_{hi}}{\sum_{i=1}^{n_h} M_{hi}}$ is the ratio of the total volume from the n_h units sampled in the h^{th} stratum, to the total acres in the n_h sample;

$\hat{Y}_{hi} = M_{hi}\bar{y}_{hi} = M_{hi} \frac{\sum_{j=1}^{m_{hi}} y_{hij}}{m_{hi}}$ is the estimated total volume of the i^{th} unit in the h^{th} stratum;

y_{hij} is the volume per acre for the j^{th} point in the i^{th} unit of the h^{th} stratum;

m_{hi} is the number of basal area points in the i^{th} unit of the h^{th} stratum; and

M_{hi} is the number of acres in the i^{th} unit of the h^{th} stratum.

The volume per acre for the i^{th} unit in the h^{th} stratum is calculated by inserting the basal area per acre into the appropriate volume-basal area prediction equation and solving the equation. The acreage of the i^{th} unit in the h^{th} stratum is known from the initial tabulation of the population by strata.

An approximate, sample-based estimate of the MSE $(\hat{Y}_R)$ (Cochran 1977, equation 11.30) is given by

$$v(\hat{Y}_R) = \sum_{h=1}^{L} \left\{ \frac{N_h^2}{n_h} (1 - f_{1h}) \frac{\sum_{i=1}^{n_h} M_{hi}^2 (\bar{y}_{hi} - \hat{\hat{Y}}_{Rh})^2}{n_h - 1} + \frac{N_h}{n_h} \sum_{i=1}^{n_h} M_{hi}^2 \frac{(1 - f_{2hi}) s_{2hi}^2}{m_{hi}} \right\} ; \qquad (2)$$

where:

N_h, n_h, M_{hi}, and m_{hi} are defined previously;

$1 - f_{1h} = \dfrac{N_h - n_h}{N_h}$ is the finite population correction factor for the sample of n_h units in the h^{th} stratum;

$1 - f_{2hi} = \dfrac{M_{hi} - m_{hi}}{M_{hi}}$ is an approximation for the finite population correction factor for the sample of basal area points in the i^{th} unit of the h^{th} stratum.

The term $1 - f_{2hi}$ is an approximation, because each basal area point varies in size. If it is assumed that, on the average, each point samples 1 acre, then there are potentially M_{hi} basal area points in the i^{th} unit of the h^{th} stratum.

This term can also be dropped if M_{hi} is assumed to be infinite.

$\bar{y}_{hi}$ is the average volume per acre for the i^{th} unit in the h^{th} stratum;

$\hat{\hat{Y}}_{Rh}$ is the average volume per acre for the h^{th} stratum;

$$\hat{\hat{Y}}_{Rh} = \frac{\sum_{i=1}^{n_h} \hat{Y}_{hi}}{\sum_{i=1}^{n_h} M_{hi}} ;$$

$s_{2hi}^2 = \sum_{j=1}^{m_{hi}} \dfrac{(y_{hij} - \bar{y}_{hi})^2}{m_{hi} - 1}$ is the variance in volume among the m_{hi} basal area points on the i^{th} unit of the h^{th} stratum; and

y_{hij} is the volume per acre for the j^{th} point on the i^{th} unit in the h^{th} stratum (defined previously).

The second term in equation (2) can be dropped when n_h/N_h is small, when the variance between subsample points is assumed negligible, or if sampling with replacement. The second term was dropped in the KPC sale estimation.

Estimator (1), $(\hat{Y}_R)$, is a biased estimator. It may have low variance if the correlation between $\hat{Y}_{hi}$ and M_{hi} is high. The problem that arises is when the strata sample sizes are small ($n \leq 4$), the bias in $\hat{Y}_R$ increases. An alternative procedure designed to reduce bias was developed by Quenouille (Cochran 1977). The Quenouille ratio estimator is recommended whenever strata sample sizes are small and separate ratio estimates are performed (Cochran 1977). The Quenouille estimators were used in the KPC sale estimations.

Keep in mind that this new cruise design utilizes stratified sampling with ratio estimates used for each stratum. The Quenouille estimators $\hat{Y}_Q$ and $v(\hat{Y}_Q)$ (Cochran 1977) will be formed as follows and will take the place of $\hat{Y}_R$ and $v(\hat{Y}_R)$ when n_h values are small, that is:

$$\hat{Y}_Q = \sum_{h=1}^{L} M_h \hat{R}_{Qh} \ . \tag{3}$$

Note the only change between equations (1) and (3) is that $\hat{R}_{Qh}$ in (3) replaces $\hat{R}_h$ in equation (1), where:

$$\hat{R}_{Qh} = n_h \hat{R}_h - (n_h - 1)\hat{\hat{R}}_h \ . \tag{3a}$$

Both estimate the average volume per acre for the h^{th} stratum.

The expression $\hat{R}_h$ is the same as defined in equation (1);

$$\hat{\hat{R}}_h = \frac{\sum_{j=1}^{n_h} \hat{R}_{hj}}{n_h} \ ; \text{ and}$$

$$\hat{R}_{hj} = \frac{\sum_{i \neq j = 1}^{n_h} \hat{Y}_{hi}}{\sum_{i \neq j = 1}^{n_h} M_{hi}} \ .$$

The $\hat{R}_{hj}$ values are formed by excluding the units one at a time from the sample, n_h, and forming $\hat{R}_{hj}$ for the remaining units, such that there will be n_h estimates of $\hat{R}_{hj}$ for each stratum.

The sample-based estimator of the variance of the Quenouille estimator is given by:

$$v(\hat{Y}_Q) = \sum_{h=1}^{L} M_h^2 \, v(\hat{R}_{Qh}) \ ; \tag{4}$$

where, $v(\hat{R}_{Qh})$ is a function of the variance among estimates of $\hat{R}_{hj}$, given by:

$$v(\hat{R}_{Qh}) = \frac{(N_h - n_h)\,(n_h - 1)}{N_h \, n_h} \sum_{j=1}^{n_h} (\hat{R}_{hj} - \hat{\hat{R}}_h)^2 \ . \tag{4a}$$

All terms have been defined previously.

Equation (4) ignores the source of variation among points within units, as discussed previously.

Reanalysis

After completion of the cruise, the cutting units that define the population may require changes for the following reasons:

1. Estimated volume from the cruise is too small or too large.

2. Changes in the EIS may result in deletion or addition of units.

3. Additional input in the negotiation process may result in deletion or addition of units.

When cutting units are deleted, they should be removed at random from each stratum. If units are removed and replaced purposively, care should be taken to insure that the average volume per acre and the variance among units within strata do not change appreciably. Units should be removed from the population without the knowledge of which units were field sampled. Information collected in the basal area sample will be dropped for specific units removed from the population.

Units added to the population after the cruise is completed should be stratified by the same procedures used in the stratification of the initial population. If the addition is a very small percentage change in the strata areas, no new field sampling should be required. But if a substantial amount of acreage is added, similar to the blowdown in the 1979-1984 KPC rate redetermination, then additional units should be sampled for basal area in the field. New units to be sampled should be drawn at random with the same intensity as in the previous sampling.

After the additions and deletions are completed, the estimation process is reanalyzed to get the new estimates. This process may be repeated several times to satisfy the EIS, the purchaser, and the Forest Service's objectives.

Adding and subtracting units from the population and reanalyzing the sample as a new stratified random sample should not appreciably affect the reliability of the estimates if the above rules are followed.

Example

The cruise design for KPC's 1984-89 redetermination was implemented in the following manner.

1. The tentative harvest unit selections on which the cruise was based consisted of 576 units encompassing 38,640 acres.

2. The harvest units were arrayed according to their respective strata ranges by determining the acreage, by type, from Forest type maps. The average volume per acre for each type, from timber inventory data, was then multiplied by the acreage of the respective types within the unit. The weighted average volume per acre was then determined for each unit. The total unit acreage was placed in the stratum coinciding with the weighted average volume per acre. The strata ranges were defined as follows:

Strata	Strata Range
	(Average MBF/acre)
1	8.00 - 16.55
2	16.56 - 25.25
3	25.26 - 37.40
4	37.41+
5 (Alaska-cedar)	(All ranges)

The Alaska-cedar stratum was formed to provide greater precision in the estimates of Alaska-cedar volume. Units were placed in this stratum by staff who were familiar with on-the-ground conditions. The intent was to isolate units having a high incidence of Alaska-cedar. Units in other strata also contained Alaska-cedar, but supposedly to a lesser degree.

3. The total sample size (n) for strata 1-4 and the allocation to the individual strata were computed by equations (7) and (8) in the appendix. Estimates of the mean volumes per acre and the strata variances were obtained from prior cruises and from timber inventory information.

Sample size for Alaska-cedar, n_5, was computed independently as follows:

$$n_5 = \frac{t^2 M_5^2 s_5^2}{SE_5^2} .$$

where:

M_5 is the total acres in the Alaska-cedar stratum,

s_5 is the standard deviation in mean volume per acre for the Alaska-cedar stratum,

SE_5 is the specified standard error for stratum 5, and

t is the Student's t value for the specified confidence probability and infinite degrees of freedom.

3a. Specifying precision requirements:

The precision requirements are established by reducing the contracted volume by the right-of-way volume and by the volume of units left over from previous periods. The residual volume in strata 1-4 and stratum 5 are multiplied by the standards specified in the objectives.

The total contracted volume was 960.0 MMBF. There were 31.7 MMBF of right-of-way volume (ROW) and residual volume left over from the previous 5-year period; thus, the 31.7 MMBF is subtracted from the 960 MMBF. The estimate of the total volume in the Alaska-cedar stratum is 116.3 MMBF. This was determined by multiplying the number of acres by an approximation of the average volume per acre. The specified standard error for the Alaska-cedar stratum was ± 20 percent at the 68 percent confidence probability.

Therefore,

$$SE_5^2 = (0.20(116.3))^2 = 541.0 \text{ MMBF}^2 .$$

The residual volume estimate for strata 1-4 is computed as follows:

Total contract volume	960.0	MMBF
ROW and previous units	- 31.7	
Cedar stratum	- 116.3	
Residual	812.0	MMBF.

The standard error for strata 1-4 was ± 10 percent at the 68 percent confidence probability.

Therefore,

$$SE_{1\text{-}4}^2 = (0.10(812.0))^2 = (81.2)^2 \text{ MMBF}^2 .$$

3b. Preliminary estimates:

Strata	Number of cutting units	Strata size	Standard deviations	Variance
		(M acres)	(MBF/acre)	(MBF/acre)^2
1	$N_1 = 61$	$M_1 = 3{,}242$	$s_1 = 12.625$	$s_1^2 = 159.391$
2	$N_2 = 239$	$M_2 = 16{,}338$	$s_2 = 12.423$	$s_2^2 = 154.331$
3	$N_3 = 161$	$M_3 = 11{,}477$	$s_3 = 19.248$	$s_3^2 = 370.486$
4	$N_4 = 38$	$M_4 = 2{,}132$	$s_4 = 13.751$	$s_4^2 = 189.090$
5	$N_5 = 77$	$M_5 = 5{,}451$	$s_5 = 17.040$	$s_5^2 = 290.362$

The values of N_h and M_h are obtained from the array of units by strata. The values of s_h were obtained from previous FBS plots, thus, are standard deviations among average volumes per acre; that is:

$$s_h^2 = \frac{\sum_{i=1}^{n_h} (\bar{y}_{hi} - \bar{\bar{y}}_h)^2}{n_h - 1} .$$

Note that the accepted standard deviations are rather large relative to the range in average volume that defines the strata boundaries. Therefore, these s_h^2 are probably conservative (that is, n_h values may be larger than necessary).

The variances required in equations (7) and (8) (see appendix) are different from those presented in the above tabulation. The variances required are the variances among total unit volumes within strata, rather than the variances among average volumes per acre. The deviations between average volumes per acre for the i^{th} unit and the average volume for the strata ($\bar{y}_{hi} - \bar{\bar{y}}_h$) need to be weighted by the square of the acres of the i^{th} unit ($M_{hi}^2 (\bar{y}_{hi} - \bar{\bar{y}}_h)$), as in equation (2).

To approximate the variances in equations (7) and (8), all values of s_h, from the tabulation, were multiplied by $\overline{M}_h$, the average size of a unit in stratum h. Because $M_h = N_h\overline{M}_h$, values of $\overline{M}_h$ were used as weights rather than those of N_h. Both should produce approximately the same result.

Normally such complexity to estimate s_h would not be required because approximations for the correct s_h would be available. If values of s_h^2 used in planning are poor estimates of the true variances, the estimates from the cruise will not be biased. Precision estimates will be greater or less than the planned levels.

3c. Computations of n_h:

Inserting estimates into the formula for the Alaska-cedar stratum (h=5) produced $n_5 = 16$; for example:

$$n_5 = \frac{29.7 \times 290.4}{541.0} = \frac{8624.9}{541.0} = 15.9 \text{ units} .$$

Inserting estimates into equation (7) produced $n_{1\text{-}4} = 18$; for example:

$$n = \frac{(40.9 + 203.0 + 220.9 + 29.3)^2}{(81.2^2 + 516.7 + 2521.5 + 4252.1 + 403.1)} = \frac{244{,}134.8}{14{,}286.8} = 17.1 \text{ units} .$$

Strata n_1, n_2, n_3, and n_4 are proportioned from equation (8) as follows:

$$n_1 = 17.1 \left(\frac{40.9}{494.1}\right) = 1.4 \text{ units;}$$

$$n_2 = 17.1 \left(\frac{203.0}{494.1}\right) = 7.0 \text{ units;}$$

$$n_3 = 17.1 \left(\frac{220.9}{494.1}\right) = 7.6 \text{ units; and}$$

$$n_4 = 17.1 \left(\frac{29.3}{494.1}\right) = 1.0 \text{ units.}$$

The resulting sample sizes for each stratum were:

Strata	Computed	Selected
	(Number of units)	(Number of units)
1	2	4
2	7	14
3	8	16
4	2	4
5	16	16
Totals	35	54

The strata sample sizes for strata 1-4 were doubled and the minimum sample size was set at 4 by Timber Management, Tongass National Forest, because this was the first trial of this design and the variance approximations were suspect.

4. Random samples were then selected according to the above allocation. The harvest units selected for visits in the field for each stratum were as follows:

Stratum and unit		Acres
Stratum 1:		
1	63-5	9
2	575-24	48
3	597-19	60
4	596-23	30
		147
Stratum 2:		
1	547-8	81
2	58-9	130
3	42-33	22
4	744-11	90
5	734-21	50
6	740-7	57
7	32-2	106
8	37-5	176
9	620-34	95
10	577-10	38
11	44-75	3
12	44-79	32
13	27-8	45
14	13-5	43
		968

Stratum and unit		Acres
Stratum 3:		
1	41-15	58
2	561-1	33
3	5461-4	73
4	534-3	102
5	531-12	86
6	531-14	100
7	531-40	60
8	60-19	44
9	11-9	41
10	54-5	74
11	736-111	138
12	36-15	50
13	619-45	53
14	620-26	85
15	620-101	46
16	44-1	65
		1,108

Stratum and unit		Acres
Stratum 4:		
1	561-20	25
2	65-21	3
3	45-27	24
4	620-11	90
		142
Stratum 5:		
1	45-34	89
2	65-18	116
3	65-5	55
4	548-2	53
5	548-10	46
6	620-51	32
7	620-96	108
8	577-6	53
9	578-1	68
10	575-10	48
11	586-4	81
12	595-15	105
13	595-26	50
14	573-1	69
15	582-1	88
16	29-50	106
		1,167

The 54 sample units encompassed 3,532 acres (1,429 ha). Basal area points were then located in each unit using a grid that resulted in one basal area point for approximately 3.5 acres (1.4 ha). For example, unit 547-8 in stratum 2 had 81 acres (33 ha); thus, on the average, it should have had 23 basal area points. A total of 1,025 basal area points were taken in the subsample of units using a basal area factor of 40. The tree count was taken by species on each point. Approximately 7,200 trees were sampled on the basal area points.

5. The basal area per acre on each point was displayed by species for all points. Twenty fall, buck, and scale plots for each species were selected—a total of 80 plots. The 20 plots per species were distributed uniformly over the basal area classes as shown in the following tabulation:

Number of trees per point	Western hemlock		Sitka spruce		Alaska-cedar		Western redcedar	
	Total points	FBS plots	Total points	FBS plots	Total points	FBS plots	Total points	FBS plots
1	95	2	263	4	100	3	99	3
2	132	2	104	4	69	3	86	3
3	171	2	62	4	46	3	78	3
4	158	2	26	4	47	3	37	3
5	117	2	12		24	3	38	3
6	119	2	3	4	9	3	27	3
7	71	2	0		7		8	
8	57	2	2		2		13	
9	29	2	0		3	2	9	
10	10		0		0		1	
11	6		0		1		1	
12	3	2	0		0		1	2
13	1		0		0		0	
14	1		0		0		0	
15	0		0		0		0	
16	0		0		0		1	

The FBS plots in each basal area class for each species were randomly selected from the total number of plots in that class. For the 95 basal area points having one hemlock tree, for example, two random numbers from 1 to 95 were generated. Each random number corresponded to a basal area point number. The points selected became FBS plots.

6. The 80 plots were visited in the field and the data for a fall, buck, and scale cruise was gathered at each plot. The 80 plots occurred in 36 of the 54 field-sampled units.

7. Regression analyses of gross and net volume per acre (board feet and cubic feet) over basal area per acre, were run. The equations, scatter plots, and other regression analysis statistics appear in figures 1-8 in the appendix. Refer to the section, "Volume Equation Construction," for details on model construction and selection.

8. The volume equations developed in step 7 were applied to the basal area per acre by species on each point developed in step 4 to produce volumes per acre by species for each point. Total unit volume by species was then calculated by combining all points in a unit according to the estimators specified previously. These unit volumes were the ingredients that went into the Quenouille estimators for estimating total volume for the population. Volume by log grade was also constructed at the unit level by applying average factors developed from the FBS plots.

9. Quenouille's estimators, equations (3a) and (4a), by individual species and the total of all species, were used to produce the volumes per acre and the variances for each stratum. An example for stratum 4 appears in table 1. Multiplying the volumes per acre and the variance by the stratum area and the square of stratum area, respectively, produces the estimated total volume by species and its variance for each stratum. This total volume by species and log grade is illustrated for stratum 4 in table 2.

10. Equations (3) and (4) were then used to combine the strata. The total volume for the population and its standard error appear in tables 3 and 4. Note that the total net Scribner board foot volume for the 576 units was 1,367 MMBF, and the amount planned for sale was 960 MMBF. The 576 units exceeded the volume of timber to be sold. The mean volume per acre of each species type or volume stratum used to approximate the total volume was less than what actually existed in the population: If representative data from the same parent population were used in the initial estimate, then the preliminary and final estimates should be much closer.

The objective is, nevertheless, to offer approximately 960 MMBF of timber for the sale. To reduce the 1,367 MMBF to 960 MMBF, units were removed from each strata according to procedures described earlier (see "Reanalysis"). Recall that there was a requirement in the Tongass Management Plan to maintain a specified proportion of acres in each of the volume classes. As units were removed, this specified proportion was monitored. It is possible to remove units and maintain the proportion among strata without adversely affecting the estimation procedures because volume strata were sampled independently.

The sample was reprocessed until the number of units left produced the 960 MMBF. All information from the units removed was discarded (except the FBS information). The collection of units left, after all adjustments, was the "final population."

If the remaining units are a subset of the original population, formed by excluding units randomly from each strata, then the final population is representative of the original "superpopulation," and the sample of FBS plots originally measured will be applicable to all basal area points remaining in the sample. If units are added to form the "final population," and FBS plots have not had the opportunity to be drawn, then consideration should be given to doing some additional sampling to represent the added portion of the "final population."

It is clear that the closer the initial estimate of volume is to the 960 MMBF, the less information is discarded or added. This promotes incentives to do a better job of stratification of cutting units and of initial estimation of the volume of the population.

11. Tables 5 and 6 illustrate the reanalysis of the population estimates of the KPC sale after removing 178 units. Note that the volume estimate was 941.4 MMBF after adjustments and that the standard error of estimate was 4.7 percent of the total.

The targeted precision was ±10 percent. By doubling the intensity of the sample and controlling the precision on the cedar stratum, the precision achieved was much greater than required. Much less sampling could have been performed to meet the ± 10 percent precision standard.

Acknowledgments

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Metric Equivalent

1 acre = 0.4047 hectare

Literature Cited

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Appendix 1

Sampling With Probability Proportional to Size

As explained earlier, probability proportional to size (pps) sampling is very efficient when units within strata have approximately the same average volume per acre but differ substantially in acreage (size). If, however, the strata sizes are subject to change by the addition or deletion of units, due to administrative decisions or natural causes, then the estimation scheme for pps sampling can become very complex. For this reason it was not used in the KPC sale cruise.

When these restrictions do not exist, pps sampling is the preferred procedure. A pps sample is drawn and estimation accomplished as follows:

Given values for n_h from the sample size calculations and the array of units by strata shown previously, draw n_h random numbers from the h^{th} stratum from the interval 1 to the total acres in the stratum. Each random number represents a specific acre in the stratum. The unit to which a specific random number applies is found by accumulating the acres of units from 1 to N_h, and locating the random number in the interval of accumulated acres. The unit the random number exists in will then identify the unit that will enter the sample.

The n_h units can be drawn with or without replacement. If drawn with replacement, the variance estimation is simplified to involve only the variation among unit means. And, as explained in the text, when n_h/N_h is small (approaches zero), the variation among points within units can be dropped even when sampling without replacement. The recommended method, therefore, is to draw n_h with replacement.

Upon completion of field measurements of the n_h units, the estimate of total volume for the h^{th} stratum is given by:

$$(\hat{Y}_{pps})_h = \frac{M_h}{n_h} \sum_{i=1}^{n_h} \left(\frac{\hat{Y}_{hi}}{M_{hi}} \right) = M_h \frac{\sum_{i=1}^{n_h} \bar{y}_{hi}}{n_h} = M_h \bar{\bar{y}}_h \ ;$$

where:

$\bar{y}_h$ is the unweighted volume per acre for the h^{th} stratum; all other terms are defined earlier.

The sample-based estimator of the true variance of $(\hat{Y}_{pps})_h$ is given by:

$$v(\hat{Y}_{pps})_h = \frac{M_h^2 \sum_{i=1}^{n_h} \left(\frac{\hat{Y}_{hi}}{M_{hi}} - (\hat{Y}_{pps})_h \right)^2}{n_h(n_h - 1)} = \frac{M_h^2 \sum_{i=1}^{n_h} (\bar{y}_{hi} - \bar{\bar{y}}_h)}{n_h (n_h - 1)} \ .$$

The combined estimates for all strata are given by:

$$\hat{Y}_{pps} = \sum_{h=1}^{L} (\hat{Y}_{pps})_h \ , \tag{5}$$

and by

$$v(\hat{Y}_{pps}) = \sum_{h=1}^{L} v(\hat{Y}_{pps})_h \ . \tag{6}$$

Sample Size Calculations

To estimate the size of samples needed for meeting the desired level of precision, preliminary estimates of the variances among units for each stratum have to be available.

Equal probability sampling.—In the equal probability sampling procedure, with the ratio to size estimator and optimum allocation, the total sample size for L strata is:

$$n = \sum_{h=1}^{L} n_h, \text{ computed as follows:}$$

$$n = \frac{\left(\sum_{h=1}^{L} N_h s_h \right)^2}{SE^2 + \sum_{h=1}^{L} N_h s_h^2} \ ; \tag{7}$$

where:

SE is the standard error of the total estimate required for all strata. This is a modification of Cochran's equation (1977, equation 5.25) for a specified SE of the total estimate. Then:

$$n_h = n \left[\frac{N_h s_h}{\sum_{h=1}^{L} N_h s_h} \right] ; \tag{8}$$

where:

$$s_h^2 = \sum_{i=1}^{n_h} \frac{M_{hi}^2 (\bar{y}_{hi} - \hat{\hat{Y}}_{Rh})^2}{n_h - 1} ;$$

$$\bar{y}_{hi} = \frac{\hat{Y}_{hi}}{M_{hi}} = \frac{\sum_{j=1}^{m_{hi}} y_{hij}}{m_{hi}} ;$$

$$\hat{\hat{Y}}_{Rh} = \frac{\sum_{i=1}^{n_h} \hat{Y}_{hi}}{\sum_{i=1}^{n_h} M_{hi}} ;$$ all of which were defined previously.

Sampling with probability proportional to size (pps).—In pps sampling the computation of sample sizes n_h is straightforward. Equation (6) is used and is given by:

$$v(\hat{Y}_{pps}) = \sum_{h=1}^{L} M_h^2 \frac{\sum_{i=1}^{n_h} \left(\frac{\hat{Y}_{hi}}{M_{hi}} - (\hat{\hat{Y}}_{pps})_h \right)^2}{n_h (n_h - 1)} ,$$

$$= \sum_{h=1}^{L} M_h^2 \frac{\sum_{i=1}^{n_h} (\bar{y}_{hi} - \bar{\bar{y}}_h)^2}{n_h (n_h - 1)} .$$

This can be simplified to:

$$v(\hat{Y}_{pps}) = \sum_{h=1}^{L} M_h^2 \left(\frac{s_h^2}{n_h} \right);$$

where s_h^2 is the variance among volumes per acre for the n_h units in each strata. The size of the individual units do not enter into the final calculations. Equations (7) and (8) are modified for computing the sample sizes by replacing the weights N_h with M_h. This is done because the estimates of s_h^2 are computed on the volumes per acre for each unit. For example:

$$n = \frac{\left(\sum_{h=1}^{L} M_h s_h \right)^2}{SE^2 + \sum_{h=1}^{L} M_h s_h^2}; \tag{7a}$$

and

$$n_h = n \left[\frac{M_h s_h}{\sum_{h=1}^{L} M_h s_h} \right]; \tag{8a}$$

where:

$$s_h^2 = \frac{\sum_{i=1}^{n_h} (\bar{y}_{hi} - \bar{\bar{y}}_h)^2}{n_h - 1}.$$

Controlling precision on two estimates.—The values of SE arise from the specified levels of precision. In the case of the KPC sale the specified precision was ±10 percent on the total volume at the 68 percent confidence probability and ±20 percent on the estimated volume of the Alaska-cedar stratum at the same confidence probability.

To control the precision on two estimates at the same time, start with the Alaska-cedar stratum ($h=5$). Compute the sample size for the cedar stratum similar to the example provided earlier. In that example, $SE_5 = 0.20\ (116.3) = 23.3$ MMBF, and $n_5 = 16$ units.

The next step is to compute the sample sizes for the remaining L-1 strata. This can be done by several methods. One method would be to compute n by equation (7) and use the allocation formula, (8), for all strata other than stratum 5. This would insure greater precision than specified for the total. Another alternative would be to specify the precision on just the remaining strata (that is, exclude stratum 5) and compute the sample size using equations (7) and (8) for the remaining strata. A third method—perhaps the most complicated yet the most accurate—would be to compute the SE for the remaining strata by removing the absolute standard error for stratum 5 (the Alaska-cedar stratum). This will result in a new specified SE for the remaining strata.

For example:

If SE $= \pm 0.10Y$ at 68 percent confidence probability, and $\hat{Y} = 941.4$ MMBF is the best preliminary estimate of Y, then $0.10(941.4) = 94.1$ MMBF is the specified absolute SE for the total. The specified SE^2 of the estimated total volume for all strata is given by:

$$SE_{1\text{-}5}^2 = (94.1)^2 = 8{,}862.3 \text{ MMBF}^2 .$$

Because n_5 is computed and there are values for N_h^2 and s_h^2, the specified standard error for strata 1-5 can be reduced by the contribution of stratum 5. This leaves the absolute SE desired for the remaining strata:

$$SE_{1\text{-}5}^2 - SE_5^2 = SE_{1\text{-}4}^2 \text{, or}$$

$$8{,}862.3 - 541.0 = 8{,}321.3 \text{ MMBF}^2 .$$

Equations (7) and (8) can then be used to calculate the n_h values for strata 1 through 4 using $SE_{1\text{-}4}^2 = 8{,}321.3$ MMBF^2.

In stratified sampling a complication arises in choosing a value for the degrees of freedom for the Student's t value. The sample size for looking up the degrees of freedom should be approximated for calculating confidence intervals by the following expression (Cochran 1977, equation 5.16):

$$n_e = \frac{\sum_{h=1}^{L} g_h s_h^2}{\sum_{h=1}^{L} \frac{g_h^2 s_h^4}{n_h - 1}} ;$$

where:

$$g_h = \frac{N_h(N_h - n_h)}{n_h} .$$

The value of n_e (estimated sample size) will lie between the minimum n_h and n.

Volume Equation Construction

Species total volume is an essential ingredient in the appraisal process. In addition, the relationship between volume per acre and basal area per acre will usually differ among species. The sampling procedure used on the cruise for the KPC sale to develop volume equations was therefore designed to produce separate equations for each of the major species.

Four different species sets of equations were produced. Approximately 20 FBS plots per species were selected from the lists of basal area points. These lists were constructed for each species based on the major species at the point. If a point is selected for use with a particular species volume equation it may also be used in other species equations, if other species exist on the point, because all trees regardless of species were FBS sampled on each point.

The lists of basal area points for each species were arrayed by basal area. A uniform sample was drawn over the range of basal area to insure equal sample information over the range of basal area. This was most easily accomplished by partitioning the range of basal areas into *r* discrete classes and drawing 20/*r* points at random from each class. If 20/*r* did not produce a whole number the extra points were spaced equally over the range of the basal area classes.

Detailed information was collected from the FBS plots on the trees selected by the specified basal area factor. Trees selected by the basal area factor were measured for d.b.h. and were then felled and scaled for defect and volume. The logs, by tree, were identified so the number of logs per MBF could be computed later. Each log was also graded.

FBS plots were summarized in the office to produce species estimates of:

- gross and net volume per acre,
- basal area per acre,
- volume by log grade,
- volume of woods defect,
- volume of scaling defect,
- number of logs per MBF, and
- average d.b.h.

Volume estimates were made in both board feet and cubic feet.

The process of developing volume-basal area equations is an exploratory one. In general, a linear relationship of volume per acre over basal area per acre works well; however, occasions may exist where nonlinear models are more appropriate. Another consideration in the development of volume-basal area equations is the conditioning of the extremes of the curves. For example, it may be desirable to condition the curve through zero or to be asymptotic to the basal area axis at a positive volume.

The model with the minimum residual sums of squares for the KPC sale was

$$y = ax^b \;;$$

where:

y is gross and net cubic and board foot volumes per acre;
a and b are coefficients; and
x is basal area per acre.

The volume-basal area equations developed are shown in figures 1-8.

Developing equations for major species should in general pose no problems. Minor species with highly variable form or defect may not be able to be estimated singly, however, and may have to be combined with another major species of similar geometric configuration.

Volume by log grade can also be investigated for a prediction equation of volume per acre expressed as a function of basal area per acre. If statistically significant equations exist, they can be used in the estimation of volume by log grades. If not, a set of average factors by species should be used to prorate the final estimates of total net and gross volumes by species into volumes by log grade. Average factors by species were used in the KPC sale cruise.

The final volume equations are applied to each basal point sampled in the field and a volume per acre by species is constructed (y_{hij}) for the j^{th} point in the i^{th} unit of the h^{th} stratum, so that all basal area points will possess volume per acre estimates.

The estimates of volume and variance in estimated volume are then formed according to equations (1) through (6).

Appendix 2
Tables and Figures

Table 1—Timber cruise summary of volume per acre by species and by grade for stratum 1, KPC sale[1/]

		Grades							
Species	Units	Peeler or Select	1	2	3	4	Utility	Cull	Total
Sitka spruce (code 098)	Gross board feet	1,150.1	880.6	728.7	233.5	0	1,030.6	423.3	4,446.9
	Net board feet	896.7	694.1	642.4	182.6	0	927.3		3,343.1
	Gross cubic feet	195.8	157.1	159.9	92.3	0	106.3	95.7	807.1
	Net cubic feet	184.4	142.8	149.0	86.5	0	70.7		633.4
Western redcedar (code 242)	Gross board feet	0	1,509.6	2,610.4	3,358.9	0	0	2,199.6	9,678.5
	Net board feet	0	1,084.1	1,816.2	2,580.6	0	0		5,480.9
	Gross cubic feet	0	321.6	535.7	994.2	0	0	581.6	2,433.2
	Net cubic feet	0	279.3	472.4	882.1	0	0		1,633.8
Western hemlock (code 263)	Gross board feet	3,641.3	2,762.9	4,837.1	2,730.2	0	3,037.5	2,570.0	19,579.2
	Net board feet	3,004.2	1,974.6	3,895.6	2,282.1	0	2,645.1		13,801.5
	Gross cubic feet	560.8	587.2	1,069.9	876.1	0	650.3	614.1	4,358.5
	Net cubic feet	528.5	520.7	960.4	809.0	0	454.8		3,273.4
Alaska cedar (code 042)	Gross board feet	925.4	4,120.5	2,600.2	1,449.6	0	0	1,035.4	10,131.0
	Net board feet	813.1	2,885.5	1,948.9	1,119.1	0	0		6,766.6
	Gross cubic feet	202.4	667.4	654.7	541.3	0	0	263.8	2,329.6
	Net cubic feet	190.7	624.2	598.3	493.9	0	0		1,907.1
All species	Gross board feet	5,716.8	9,273.7	10,776.5	7,772.2	0	4,068.1	6,228.2	43,835.6
	Net board feet	4,714.0	6,638.3	8,303.2	6,164.4	0	3,572.4		29,392.2
	Gross cubic feet	958.9	1,733.4	2,420.3	2,504.0	0	756.6	1,555.2	9,928.3
	Net cubic feet	903.7	1,566.9	2,180.1	2,271.5	0	525.5		7,447.8

[1/]Supporting information: Long-term sale 1984-89, number 01042; Forest number 5; Region 10. Stratum 1: 4 units were measured; numbers 199, 366, 440, and 458 contained 9, 48, 30, and 60 acres, respectively; 57 units were not measured; 61 units contained 3,242 acres.

Table 2—Timber cruise summary of total volume in thousand feet by species and by grade for stratum 1, KPC sale[1/]

Species	Units	Total volume by grade: Peeler or Select	1	2	3	4	Utility	Cull	Total
Sitka spruce (code 098)	Gross board feet	3,728.7	2,855.0	2,362.6	757.2	0	3,341.2	1,372.2	14,416.9
	Net board feet	2,907.0	2,250.3	2,082.8	592.0	0	3,006.3		10,838.4
	Gross cubic feet	634.7	509.3	518.5	299.3	0	344.5	310.1	2,616.5
	Net cubic feet	597.9	462.8	483.0	280.3	0	229.3		2,053.4
Western redcedar (code 242)	Gross board feet	0	4,894.3	8,462.9	10,889.5	0	0	7,131.0	31,377.6
	Net board feet	0	3,514.5	5,888.3	8,366.2	0	0		17,769.0
	Gross cubic feet	0	1,042.7	1,736.7	3,223.3	0	0	1,885.7	7,888.4
	Net cubic feet	0	905.5	1,531.7	2,859.7	0	0		5,296.8
Western hemlock (code 263)	Gross board feet	11,805.0	8,957.5	15,682.0	8,851.3	0	9,847.7	8,332.0	63,475.7
	Net board feet	9,739.6	6,401.6	12,629.4	7,398.6	0	8,575.3		44,744.6
	Gross cubic feet	1,818.0	1,903.8	3,468.7	2,840.5	0	2,108.3	1,990.9	14,130.1
	Net cubic feet	1,713.5	1,688.1	3,113.6	2,622.7	0	1,474.5		10,612.4
Alaska cedar (code 042)	Gross board feet	3,000.0	13,358.7	8,429.8	4,699.7	0	0	3,356.6	32,844.8
	Net board feet	2,636.0	9,354.9	6,318.4	3,628.1	0	0		21,937.4
	Gross cubic feet	656.1	2,163.8	2,122.6	1,754.8	0	0	855.1	7,552.5
	Net cubic feet	618.4	2,023.6	1,939.6	1,601.4	0	0		6,183.0
All species	Gross board feet	18,533.9	30,065.5	34,937.3	25,197.6	0	13,188.9	20,191.9	142,115.0
	Net board feet	15,282.7	21,521.4	26,918.8	19,984.9	0	11,581.6		95,289.4
	Gross cubic feet	3,108.8	5,619.6	7,846.6	8,117.8	0	2,452.8	5,041.9	32,187.6
	Net cubic feet	2,929.8	5,080.0	7,067.9	7,364.1	0	1,703.8		24,145.7

[1/]Supporting information: Long-term sale 1984-89, number 01042; Forest number 5; Region 10. Stratum 1: 4 units were measured; numbers 199, 366, 440, and 458 contained 9, 48, 30, and 60 acres, respectively; 57 units were not measured; 61 units contained 3,242 acres.

Table 3—Timber cruise summary of total volume, woods defect, scaling defect, and other characteristics for 576 units in the 1984-89 recruise of the KPC sale[1/]

Item	Estimates
Scribner volume:	
Gross (MMBF)	1,896.7
Net (MMBF)	1,367.4
Net per acre (BF)	35,388.4
Cubic volume:	
Gross (MMCF)	412.6
Net (MMCF)	310.7
Net per acre (CF)	8,043.0
Woods defect (percent):	
Scribner	27.9
Cubic	24.7
Scaling defect (percent):	
Scribner	15.8
Cubic	11.0
Number of logs per thousand board feet	5.3
Number of logs per cunit	2.5
Number of trees per acre	125.9
Average d.b.h. (inches)	19.3

[1/]Supporting information: Long-term sale 1984-89, number 01042; Forest number 5; Region 10.

Table 4—Timber cruise summary of estimated total volume and the standard error for 576 units in the 1984-89 recruise of the KPC sale[1/]

Item	Scribner volume				Cubic volume			
	Per acre	Total	Standard error		Per acre	Total	Standard error	
	Board feet	- - Million board feet - -		Percent	Cubic feet	- Million cubic feet -		Percent
Gross	49,087.7	1,896.7	--	--	10,680.6	412.7	--	--
Net	35,388.4	1,367.4	59.6	4.4	8,042.6	310.8	10.4	3.4

1/Supporting information: Long-term sale 1984-89, number 01042; Forest number 5; Region 10.

Table 5—Timber cruise summary of volume, woods defect, scaling defect, and other characteristics for 398 units in the 1984-89 recruise of the KPC sale[1/]

Item	Estimates
Scribner volume:	
Gross (MMBF)	1,299.2
Net (MMBF)	941.3
Net per acre (BF)	36,178.0
Cubic volume:	
Gross (MMCF)	292.16
Net (MMCF)	212.3
Net per acre (CF)	8,159.0
Woods defect (percent):	
Scribner	27.5
Cubic	24.8
Scaling defect (percent):	
Scribner	15.2
Cubic	10.9
Number of logs per thousand board feet	5.6
Number of logs per cunit	2.6
Number of trees per acre	131.4
Average d.b.h. (inches)	19.4

1/Supporting information: Long-term sale 1984-89, number 01042; Forest number 5; Region 10.

Table 6—Summary of estimated total volume and the standard error for 398 units in the 1984-89 recruise of the Ketchikan Sale[1]

Item	Scribner volume				Cubic volume			
	Per acre	Total		Standard error	Per acre	Total	Standard error	
	Board feet	- Million board feet -		Percent	Cubic feet	- Million cubic feet -		Percent
Gross	49,930.1	1,299.3	--	--	10,843.9	108.4	--	--
Net	36,177.9	941.4	44.6	4.7	8,159.2	212.3	7.7	3.6

[1]Supporting information: Long-term sale 1984-89, number 01042; Forest number 5; Region 10.

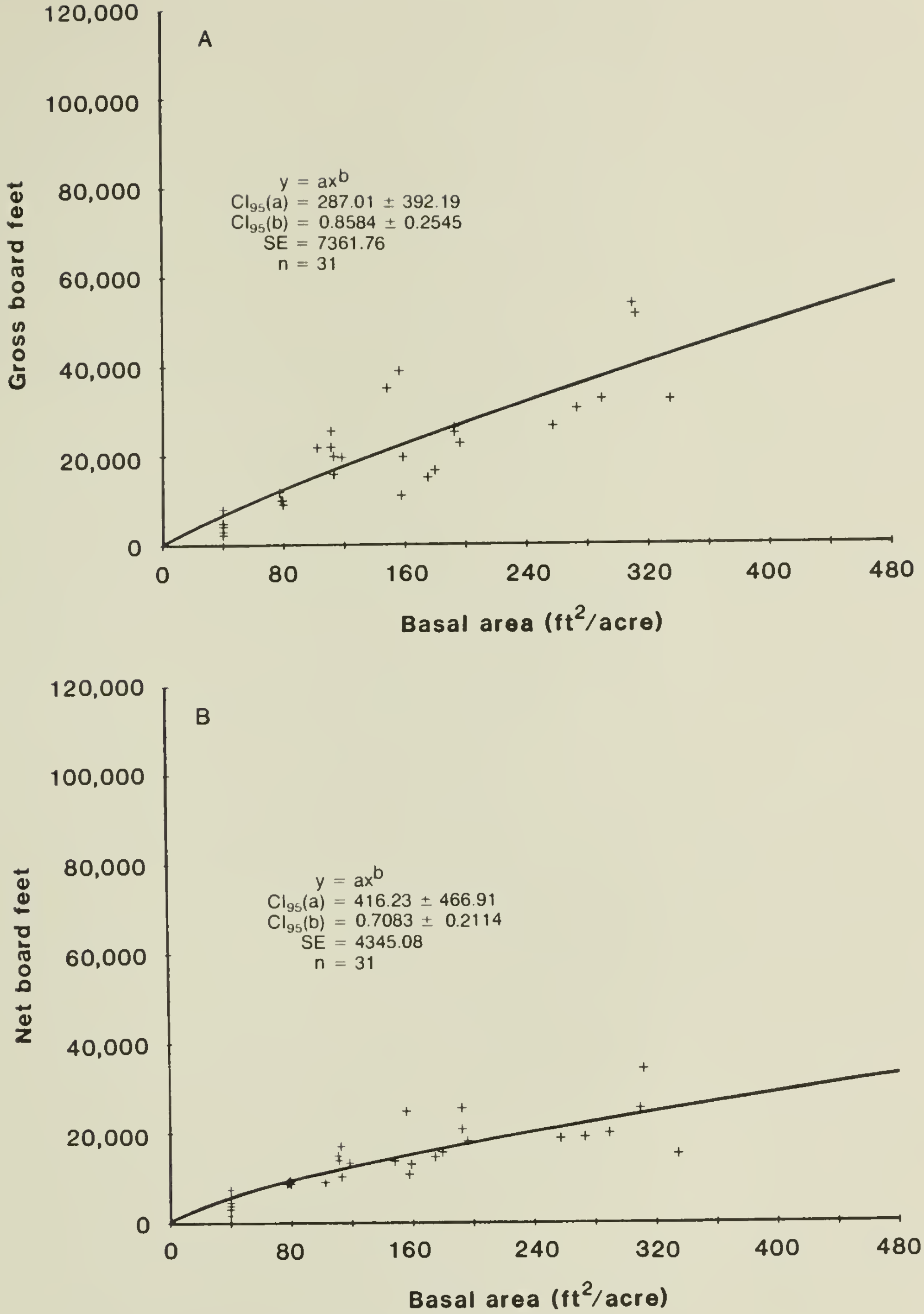


Figure 1.—Regressions of (A) gross board foot volume per acre and (B) net board foot volume per acre of Alaska-cedar (species 042) over basal area per acre for the KPC sale.

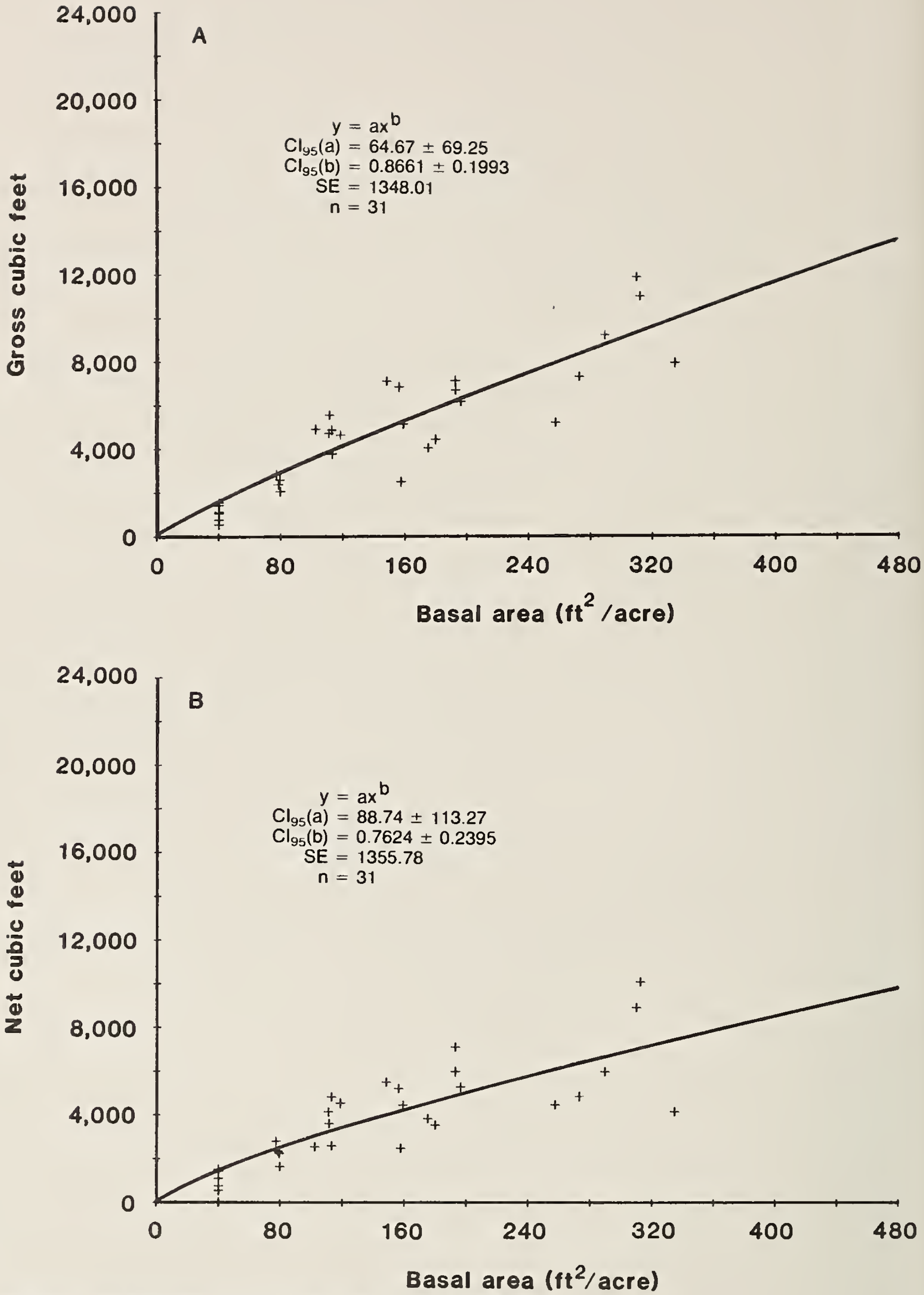


Figure 2.—Regressions of (A) gross cubic foot volume per acre and (B) net cubic foot volume per acre of Alaska-cedar (species 042) over basal area per acre for the KPC sale.

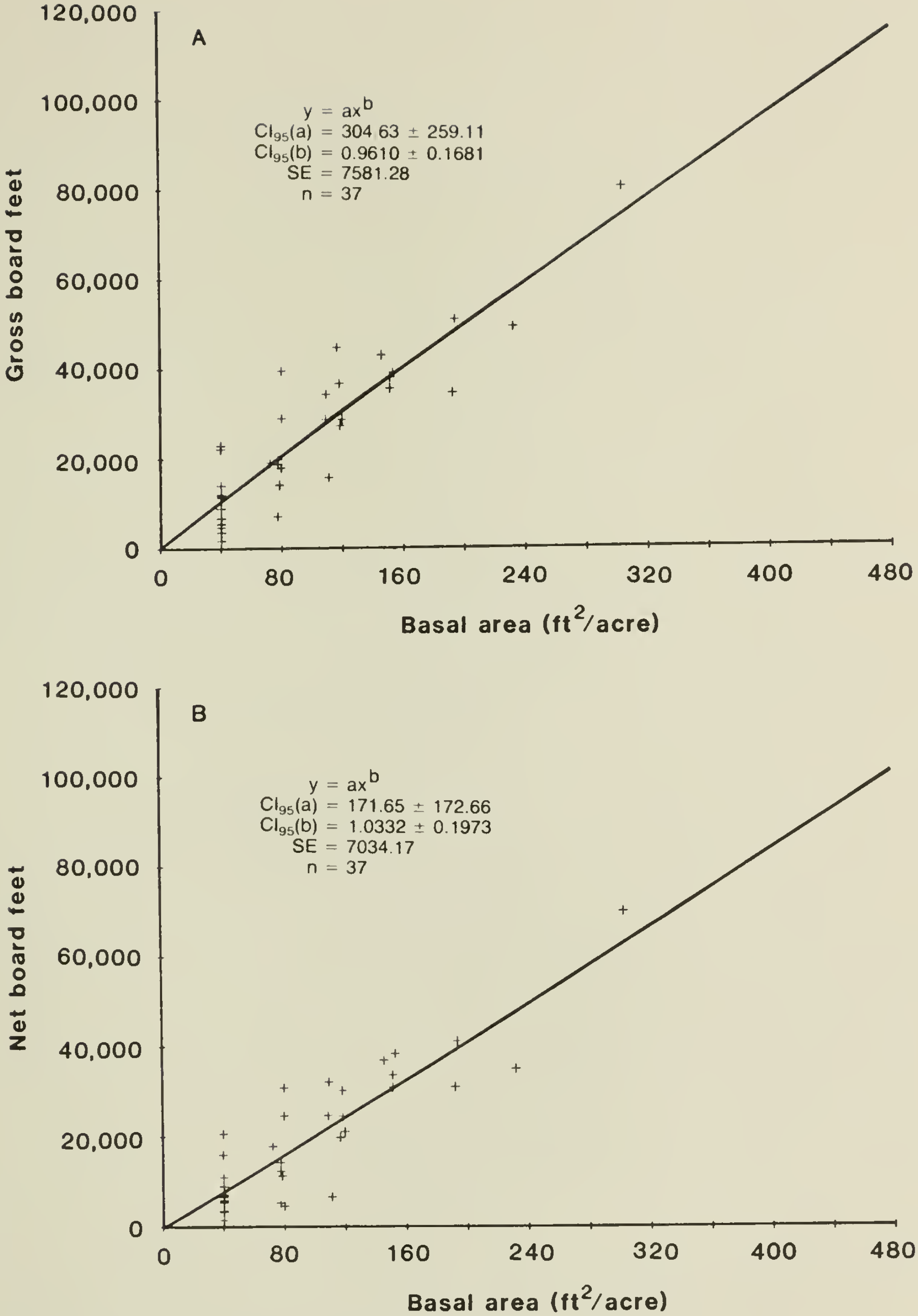


Figure 3 —Regressions of (A) gross board foot volume per acre and (B) net board foot volume per acre of Sitka spruce (species 098) over basal area per acre for the KPC sale.

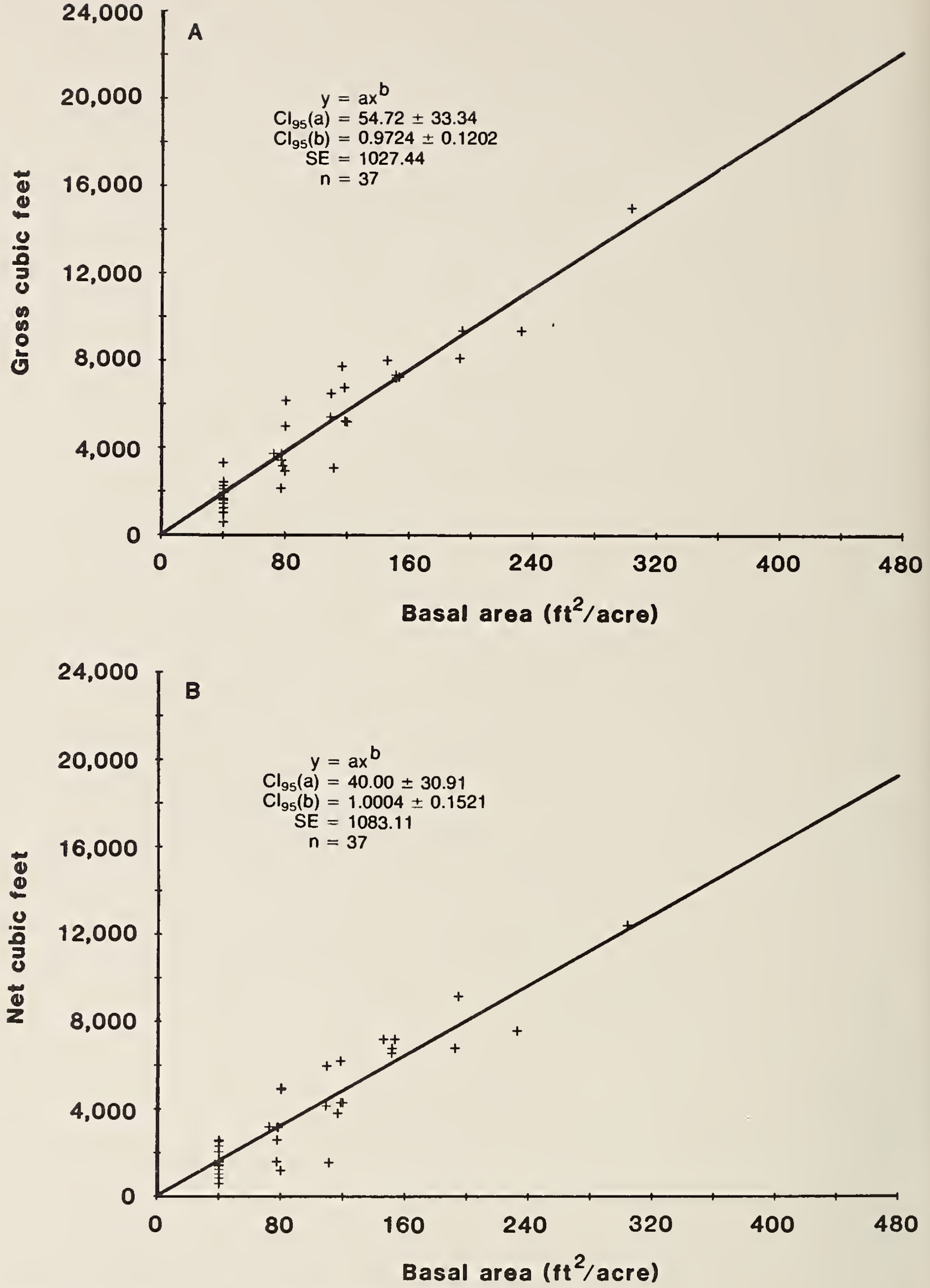
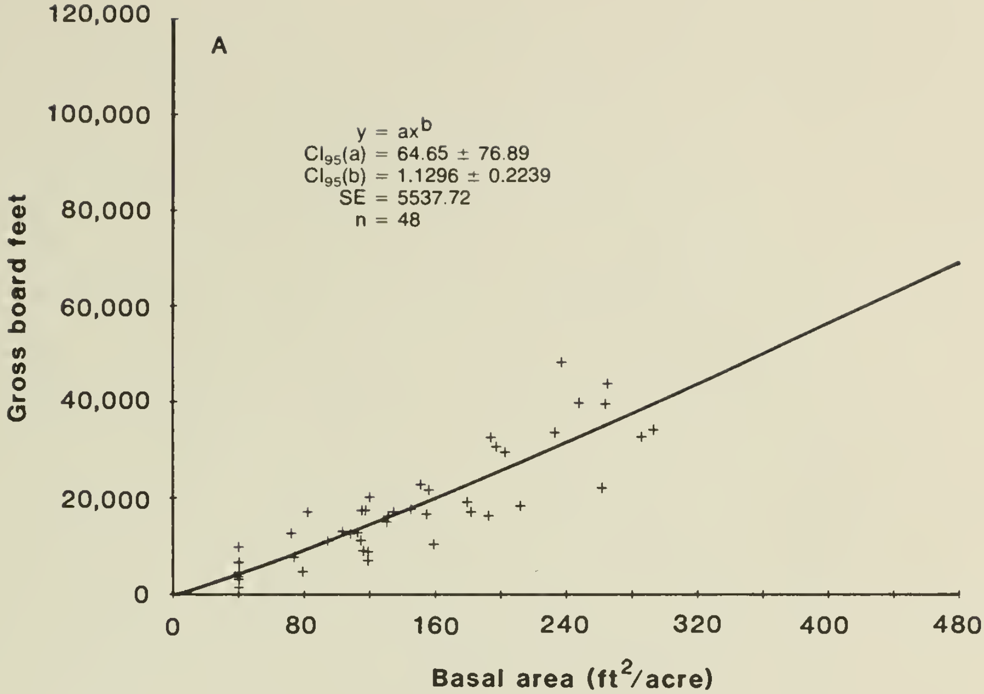


Figure 4.—Regressions of (A) gross cubic foot volume per acre and (B) net cubic foot volume per acre of Sitka spruce (species 098) over basal area per acre for the KPC sale.



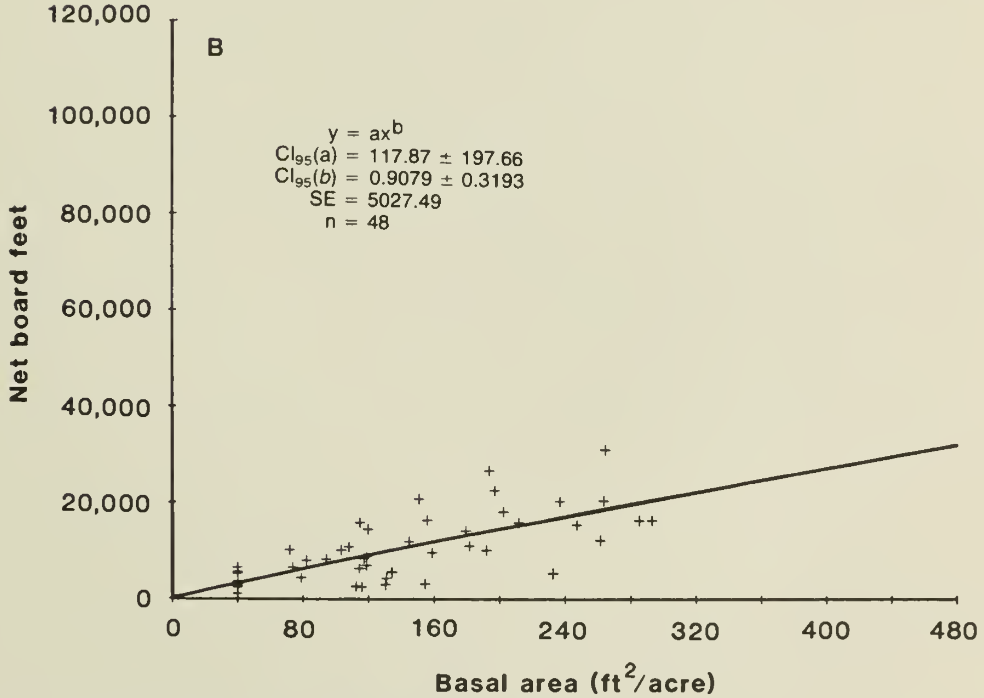
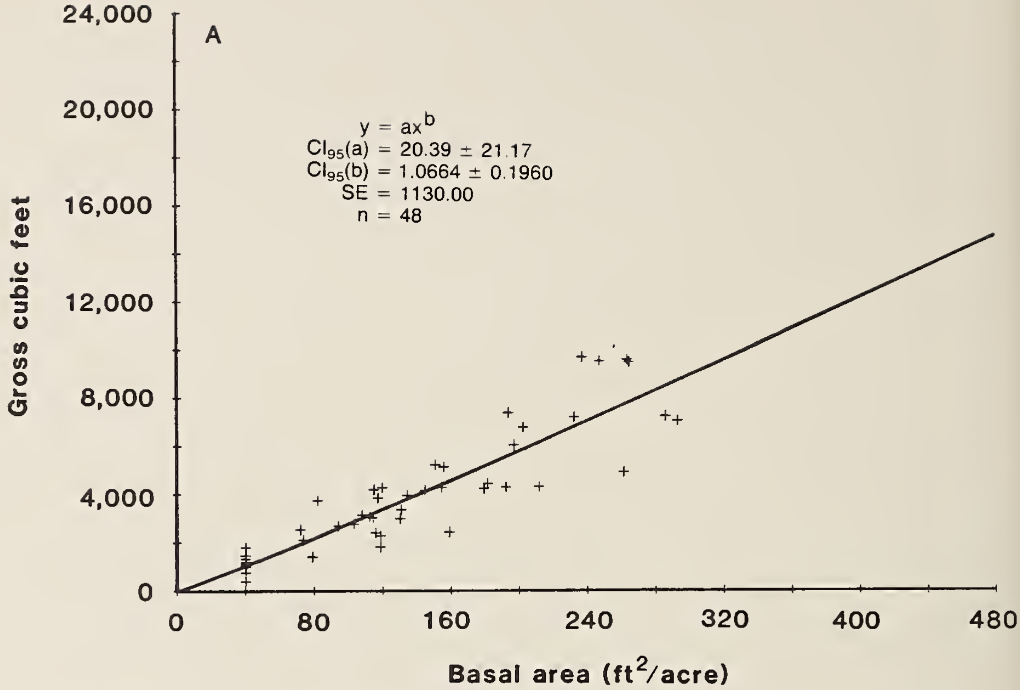


Figure 5.—Regressions of (A) gross board foot volume per acre and (B) net board foot volume per acre of western red-cedar (species 242) over basal area per acre for the KPC sale.



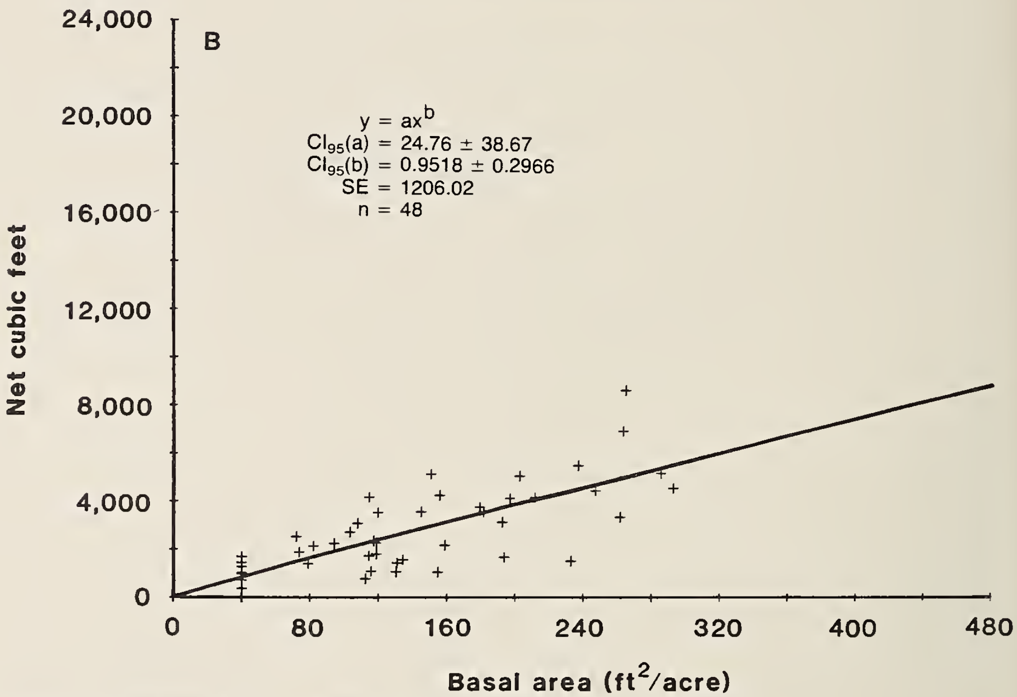


Figure 6.—Regressions of (A) gross cubic foot volume per acre and (B) net cubic foot volume per acre of western red-cedar (species 242) over basal area per acre for the KPC sale.

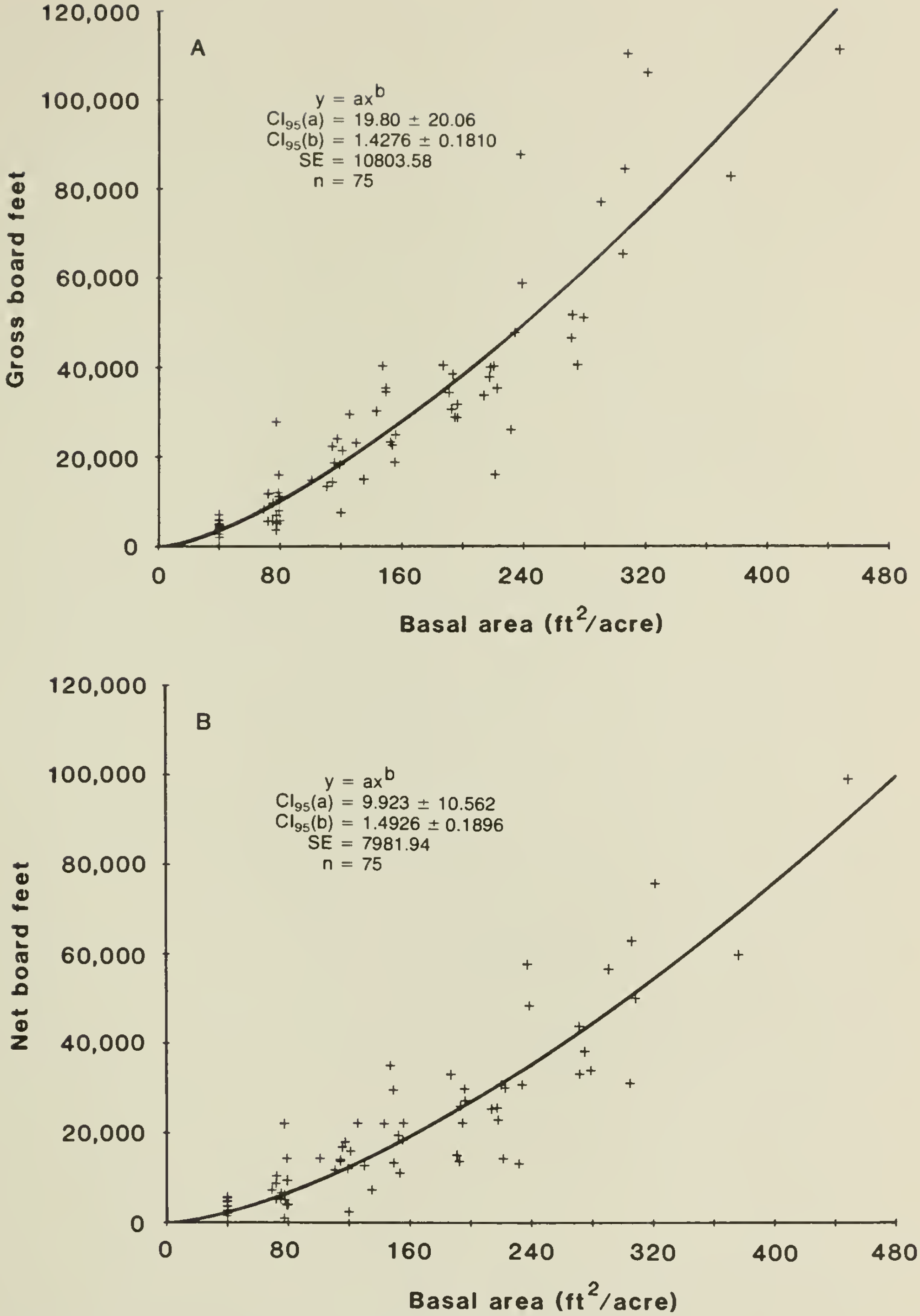


Figure 7.—Regressions of (A) gross board foot volume per acre and (B) net board foot volume per acre of western hemlock (species 263) over basal area per acre for the KPC sale.

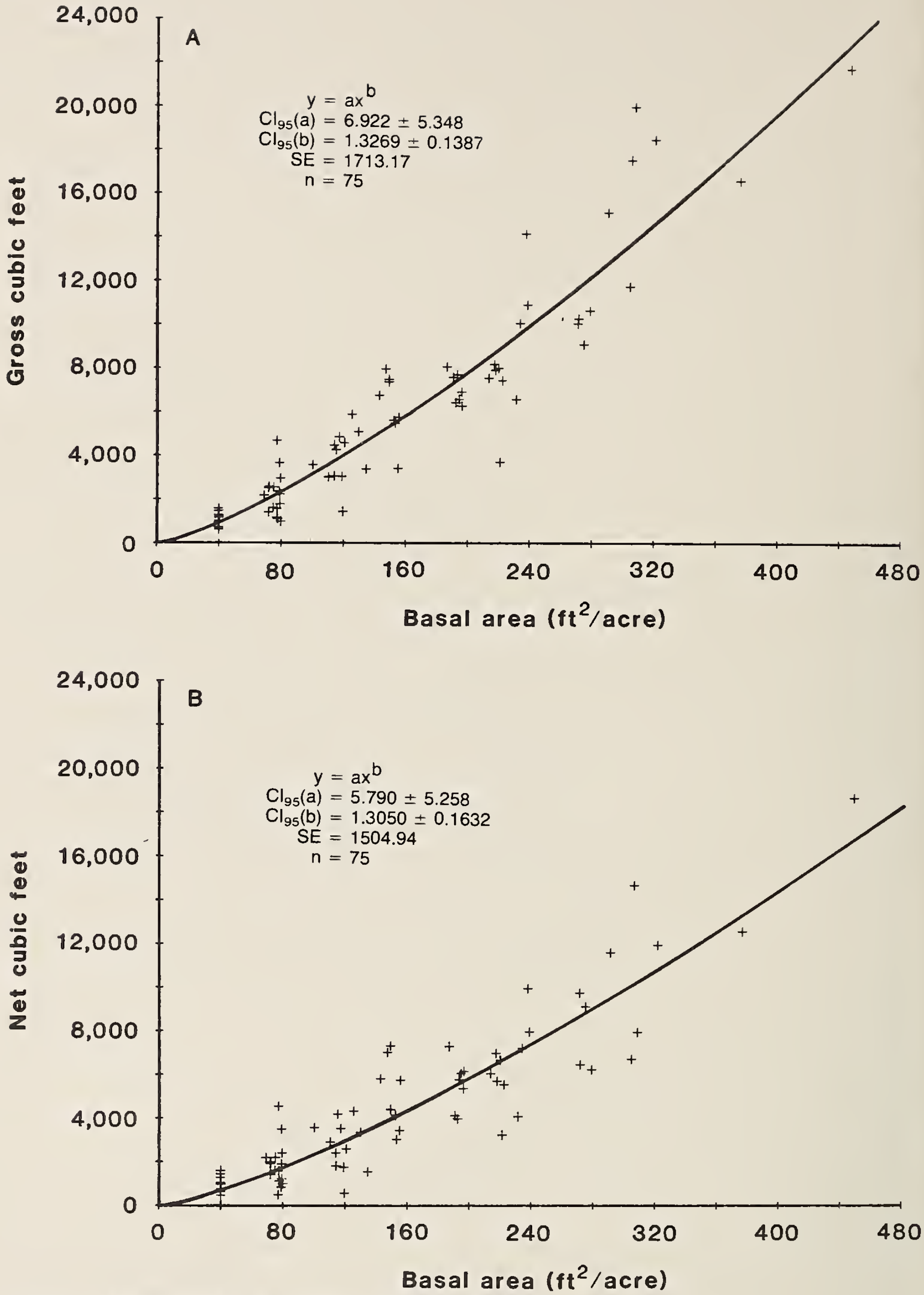


Figure 8.—Regressions of (A) gross cubic foot volume per acre and (B) net cubic foot volume per acre of western hemlock (species 263) over basal area per acre for the KPC sale.

Pacific Northwest Forest and Range
Experiment Station
319 S.W. Pine St.
P.O. Box 3890
Portland, Oregon 97208